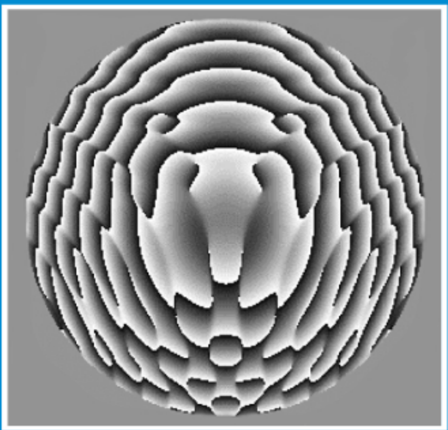
WOODHEAD PUBLISHING SERIES IN ELECTRONIC AND OPTICAL MATERIALS



Computer design of diffractive optics

Edited by V. A. Soifer

CISP

WOODHEAD PUBLISHING

Woodhead Publishing Series in Electronic and Optical Materials:
Number 50

Computer design of diffractive optics

Edited by V. A. Soifer

Cambridge International Science Publishing Limited
in association with
Woodhead Publishing Limited

WP
WOODHEAD PUBLISHING

Oxford Cambridge Philadelphia New Delhi

CISP

Published by Cambridge International Science Publishing Limited in association with Woodhead Publishing Limited

Cambridge International Science Publishing Limited, 7 Meadow Walk, Great Abington, Cambridge CB21 6AZ, UK
www.cisp-publishing.com

Woodhead Publishing Limited, 80 High Street, Sawston, Cambridge CB22 3HJ, UK
www.woodheadpublishing.com; www.woodheadpublishingonline.com

Woodhead Publishing, 1518 Walnut Street, Suite 1100, Philadelphia, PA 19102-3406, USA

Woodhead Publishing India Private Limited, G-2, Vardaan House, 7/28 Ansari Road, Daryaganj, New Delhi – 110002, India

First published 2013, Cambridge International Science Publishing Limited and Woodhead Publishing Limited. This work is a revised and translated version of an original Russian-language version published in 2009 by Fizmatlit, Russia.

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library.

Library of Congress Control Number: 2012948938

ISBN 978-1-84569-635-1 (print)
ISBN 978-0-85709-374-5 (online)
ISSN 2050-1501 Woodhead Publishing Series in Electronic and Optical Materials (print)
ISSN 2050-151X Woodhead Publishing Series in Electronic and Optical Materials (online)

The publisher's policy is to use permanent paper from mills that operate a sustainable forestry policy, and which has been manufactured from pulp which is processed using acid-free and elemental chlorine-free practices. Furthermore, the publisher ensures that the text paper and cover board used have met acceptable environmental accreditation standards.

Typeset by Butterfly Info Services, India
Printed and bound in the UK by the MPG Books Group

Contents

8 Analysis of transverse modes of laser radiation 515

Preface

In 2000, the publishing house Fizmatlit, Moscow, published the book Methods of Computer Optics, translated into English in 2002 and published by John Wiley & Sons, New York. A second, revised edition was published in 2003, which shows the interest from readers.

During this period, the main authors of the first edition expressed a critical approach to the presentation of some of the earlier chapters written by them and decided to improve them. Secondly, a lot of new research results in the field of electromagnetic methods of analysis and synthesis of diffractive optical elements with greater functionality and applications in laser systems appeared.

These reasons have prompted the authors to write a new book, based on the material of the previous editions, with expansion of the sections devoted to the solution of direct and inverse problems of diffraction theory. At the same time, given the limited scope of the content of the new book, many application issues, in particular issues of technology for diffractive optical elements, are described only concisely.

The book consists of 11 chapters, united by the idea of computer synthesis of diffractive optical elements with a highly functional conversion of laser radiation and the properties of the wave fields resulting from such changes.

The book was written by the Institute of Image Processing Systems, Russian Academy of Sciences: Chapter 1 – D.L. Golovashkin, V.V. Kotlyar, Chapter 2 – V.A. Soifer, Chapter 3 – L.L. Doskolovich, N.L. Kazan, V.A. Soifer, Chapter 4 – V.V. Kotlyar, L. Doskolovich, V.A. Soifer, Chapter 5 – L.L. Doskolovich, V.A. Soifer, Chapter 6 – L.L. Doskolovich, Chapter 7 – D.L. Golovashkin, V.V. Kotlyar, Chapter 8 – V.S. Pavelieva, S.N. Khonina, V.V. Kotlyar, V.A. Soifer, Chapter 9 – S.N. Khonina, V.V. Kotlyar, V.A. Soifer, Chapter 10 – R.V. Skidanova, S.N. Khonina, V.V. Kotlyar, V.A. Soifer, Chapter 11 – V.S. Pavelieva, D.L. Golovashkin, V.A. Soifer.

The book is based on research carried out in the framework of the Russian–American program 'Basic Research and Higher Education' (grant CRDF REC-SA-014-02), Basic Research (grants 04-01-96517, 04-07-90149, 04 -02-08094, 05-01-08020, 05-08-50298, 05-08-04033), the President of the Russian Federation grant (MD-209.2003.01, 210.2003.01 MD, MD-4003.2005.09, MK-2568.2005.09 , SS-1007.2003.01).

The authors would like to thank:

– D.V. Nesterenko, the Laboratory of Laser Measurements, The Image Processing Systems Institute of the Russian Academy of Sciences for, conducting computer simulations;

– Members of the Institute of General Physics, Russian Academy of Sciences, V.I. Konov and V.V. Kononenko for materials for chapter 11 of this book;

– Foreign partners – FIAT Research Centre (the group of Dr. P. Perlo), the Institute of Applied Optics, Friedrich-Schiller University, Jena, Germany (teams of Prof. R. Kovarshik and Dr. M. Dyuppare), University of Joensuu, Finland (group of Prof. J. Turunen) for assistance in preparing the samples and the joint DOE experimental studies;

The book is based on original research papers published in the last 25 years by co-authors, and other included Academician A.M. Prokhorov, Prof. I.N. Sisyakin, M.A. Golub, A. Volkov, S.V. Karpeev and V.A. Danilov.

1

Main equations of diffraction theory

1.1. Maxwell equations

1.1.1. Mathematical concepts and notations

The Hamiltonian operator in the Cartesian coordinate system is determined as follows: $\nabla = \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z}$

where $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are the unit vectors of the Cartesian coordinate system. The operators grad, div, rot and Δ are defined as follows:

$$\operatorname{grad} f \equiv \nabla f = \mathbf{e}_x \frac{\partial f}{\partial x} + \mathbf{e}_y \frac{\partial f}{\partial y} + \mathbf{e}_z \frac{\partial f}{\partial z}$$

$$\operatorname{div} \mathbf{F} \equiv (\nabla, \mathbf{F}) = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\operatorname{rot} \mathbf{F} \equiv [\nabla, \mathbf{F}] = \det \begin{pmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{pmatrix}$$

$$\Delta f \equiv \nabla^2 f \equiv \operatorname{div} \operatorname{grad} f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\Delta \mathbf{F} \equiv \nabla^2 \mathbf{F} = \mathbf{e}_x \nabla^2 F_x + \mathbf{e}_y \nabla^2 F_y + \mathbf{e}_z \nabla^2 F_z$$

where f, $\mathbf{F} = (F_x, F_y, F_z)$ are the scalar and vector functions, (.,.) and [.,.] are the operations of the scalar and vector products, det A is the determinant of the matrix A.

The most important integral relationships of vector analysis

The Ostrogradskii–Gauss theorem:

$$\int_V \operatorname{div} \mathbf{F} dv = \oint_S (\mathbf{F}, \mathbf{n}) dS$$

where **n** is the unit vector of the external normal, *V* is the domain of the space, restricted by the surface *S*.

The Stokes theorem

$$\int_S (\operatorname{rot} \mathbf{F}, dS) = \oint_L (\mathbf{F}, d\mathbf{l})$$

Here *L* is the contour, restricting the surface S.

1.1.2. Maxwell equations in the differential form

The electromagnetic theory of light is based on the system of Maxwell equations [1] (in the Gauss unit system):

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j} + \frac{4\pi}{c}\mathbf{j}_{\text{sec}} \tag{1.1}$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{1.2}$$

$$\operatorname{div}\mathbf{D} = 4\pi\rho \tag{1.3}$$

$$\operatorname{div}\mathbf{B} = 0 \tag{1.4}$$

The notations used here and in the rest of the book are presented in Table 1.1.

Table 1.1. Electromagnetic quantities

Parameter	Notation
Charge	q
Current	I
Charge density	ρ
Current density	j
Specific conductivity	σ
Electrical vector	**E**
Magnetic vector	**H**
Electrical bias	**D**
Magnetic induction	**B**
Dielectric permittivity	ε
Magnetic permeability	μ
Velocity of light in vacuum	c

The functions **E** = **E** (**r**, t), **H** = **H** (**r**, t), **D** = **D** (**r**, t), **B** = **B** (**r**, t) describe the electromagnetic field in the medium characterised by the parameters $\varepsilon = \varepsilon$ (**E**, **r**, t), $\mu = \mu$ (**H**, **r**, t), ρ = (**r**, t), **j** = **j** (**E**, **r**, t) (r are the spatial coordinates, t is time) and secondary current $\mathbf{j}_{sec}$ which will be described separately.

Assuming that the processes in the medium are local and inertialess (the state at every point is independent of the adjacent points and at every moment of time is independent of 'prior history'), the characteristics of the field and the medium are be linked by material equations [1]:

$$\mathbf{D} = \varepsilon\mathbf{E}, \tag{1.5}$$

$$\mathbf{B} = \mu\mathbf{H}, \tag{1.6}$$

$$\boldsymbol{j} = \sigma\mathbf{E}, \tag{1.7}$$

and by the law of charge conservation

$$\mathrm{div}\mathbf{j} = -\frac{\partial\rho}{\partial t} \tag{1.8}$$

Further, it is assumed that the parameters of the medium are independent of the vectors of the field and do not change with time: $\varepsilon = \varepsilon$ (**r**), $\mu = \mu$ (**r**) (linear medium), and are scalar (isotropic medium).

If the strength of the electrical and magnetic fields can be described in the form: **E** = Re(**E** exp($-i\omega t$)), **H** = Re (**H** exp($-i\omega t$)), where **E** =**E** (**r**), **H** = **H**(**r**) are the complex-valued functions [1], ω is cyclic frequency, i is the apparent unity, we are concerned with a monochromatic field for which the equations (1.1), (1.2) take the following form:

$$\mathrm{rot}\mathbf{H} = -ik_0\dot{\varepsilon}\mathbf{E} \tag{1.9}$$

$$\mathrm{rot}\mathbf{E} = ik_0\mu\mathbf{H} \tag{1.10}$$

where $\dot{\varepsilon} = \varepsilon - i\dfrac{\sigma}{\omega}$, $k_0 = \dfrac{\omega}{c} = \dfrac{2\pi}{\lambda}$ is the wavenumber.

1.1.3. Maxwell integral equations

Integrating the equations (1.1), (1.2) with respect to the surface S restricted by contour L and accepting the Stokes theorem, we obtain the following equations:

$$\oint_L (\mathbf{H}, d\mathrm{l}) = \frac{1}{c}\frac{d}{dt}\int_S (\mathbf{D}, dS) + \frac{4\pi}{c}I, \tag{1.11}$$

$$\oint_L (\mathbf{E}, d\mathrm{l}) = -\frac{1}{c}\frac{\mathrm{d}}{\mathrm{d}t}\int_S (\mathbf{B}, d\mathbf{S}) \tag{1.12}$$

Equations (1.3), (1.4) are integrated with respect to volume V restricted by the surface S. Subsequently, using the Ostrogradskii–Gauss theorem:

$$\oint_S (\mathbf{D},\mathbf{n})\,\mathrm{d}s = 2\pi q \tag{1.13}$$

$$\oint_S (\mathbf{B},\mathbf{n})\,\mathrm{d}S = 0 \tag{1.14}$$

The system (1.11)–(1.14) is referred to as the Maxwell integral equations in the integral form.

1.1.4. Boundary conditions

Using the Maxwell integral equations for the infinitely small contours and volumes at the interface of two media, we obtain the following boundary conditions [1) for the characteristics of the electromagnetic field:

$$((\mathbf{D}_1 - \mathbf{D}_2),\mathbf{e}_y) = 4\pi\xi, \tag{1.15}$$

$$((\mathbf{E}_1 - \mathbf{E}_2),\, \mathbf{e}_z) = 0, \tag{1.16}$$

$$((\mathbf{B}_1 - \mathbf{B}_2),\, \mathbf{e}_y) = 0, \tag{1.17}$$

$$((\mathbf{H}_1 - \mathbf{H}_2),\, \mathbf{e}_z) = 4\pi(\eta,\, \mathbf{e}_x)/c, \tag{1.18}$$

where $\xi = \lim\limits_{\Delta S \to 0} \dfrac{\Delta q}{\Delta S}$ is the density of the surface charge, $\eta = \lim\limits_{\Delta l \to 0} \mathbf{e}_x \dfrac{\Delta I}{\Delta l}$ is the density of surface current (plane, separating the media 1 and 2, is normal to the vector $\mathbf{e}_y$).

1.1.5. Poynting theorem

The equation (1.1) is multiplied by **E** and equation (1.2) by **H** and, consequently, we obtain:

$$(\mathbf{E}, \operatorname{rot}\mathbf{H}) = \frac{1}{c}\left(\mathbf{E}, \frac{\partial \mathbf{D}}{\partial t}\right) + \frac{4\pi}{c}(\mathbf{E},\mathbf{j})$$

$$(\mathbf{H}, \operatorname{rot}\mathbf{E}) = -\frac{1}{c}\left(\mathbf{H}, \frac{\partial \mathbf{B}}{\partial t}\right)$$

Deducting the second equation from the first one, we obtain the Poynting theorem [1] according to which:

$$\operatorname{div}[\mathbf{E},\mathbf{H}] = -\frac{1}{c}\left(\left(\mathbf{H}, \frac{\partial \mathbf{B}}{\partial t}\right) - \left(\mathbf{E}, \frac{\partial \mathbf{D}}{\partial t}\right)\right) - \frac{4\pi}{c}(\mathbf{j},\mathbf{E}) \tag{1.19}$$

In the integral form

$$\frac{c}{4\pi}\oint\left(\left[\mathbf{E},\mathbf{H}\right],\mathbf{n}\right)dS=-\frac{1}{4\pi}\int_V\left(\left(\mathbf{H},\frac{\partial\mathbf{B}}{\partial t}\right)+\left(\mathbf{E},\frac{\partial\mathbf{D}}{\partial t}\right)\right)dV-\int_V\left(\mathbf{j},\mathbf{E}\right)dV \quad (1.20)$$

we have the balance equation of the energy of the electromagnetic field in the volume V. The energy in the volume V is $W=\frac{1}{8\pi}\int_V\left(\left(\mathbf{H},\mathbf{B}\right)+\left(\mathbf{E},\mathbf{D}\right)\right)dV$, the consumed power $P=\int_V\left(\mathbf{j},\mathbf{E}\right)dV$ and $\mathbf{\Pi}=\frac{c}{4\pi}[\mathbf{E},\mathbf{H}]$ is the Umov–Poynting vector, indicating the direction of movement of energy and is equal to the density of the energy flux.

The monochromatic field can be described by the Umov–Poynting complex vector

$$\mathbf{\Pi}=\frac{c}{8\pi}[\mathbf{E},\mathbf{H}^*]$$

where the symbol * indicates the complex conjugation, and the mean value of the Umov–Poynting vector is equal to the real part of the complex vector.

1.2. Differential equations in optics

1.2.1. Wave equations

The current and charges, usually not found in optics problems, are excluded from the Maxwell equations. Consequently, equations (1.1) and (1.2) take the following form:

$$\operatorname{rot}\mathbf{H}=\frac{\varepsilon}{c}\frac{\partial\mathbf{E}}{\partial t} \quad (1.21)$$

$$\operatorname{rot}\mathbf{E}=\frac{\mu}{c}\frac{\partial\mathbf{H}}{\partial t} \quad (1.22)$$

Dividing both parts of equation (1.22) by μ and using the rot operator:

$$\operatorname{rot}\left(\frac{1}{\mu}\operatorname{rot}\mathbf{E}\right)+\frac{1}{c}\operatorname{rot}\frac{\partial\mathbf{H}}{\partial t}=0 \quad (1.23)$$

Equation (1.21) is differentiated with respect to time to exclude the second term from equation (1.23):

$$\operatorname{rot}\left(\frac{1}{\mu}\operatorname{rot}\mathbf{E}\right)+\frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0$$

Consequently, taking into account that

$\mathrm{rot}\,\alpha\mathbf{u} = \alpha\,\mathrm{rot}\,\mathbf{u} + \left[\mathrm{grad}\ \alpha, \mathbf{u}\right]$ and $\mathrm{rot}\ \mathrm{rot}\,\mathbf{u} = \mathrm{grad}\ \mathrm{div}\,\mathbf{u} - \nabla^2\mathbf{u}$

we obtain:

$$\nabla^2\mathbf{E} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + [\mathrm{grad}\ (\ln\mu), \mathrm{rot}\,\mathbf{E}] - \mathrm{grad}\ \mathrm{div}\,\mathbf{E} = 0 \tag{1.24}$$

For the equation div (εE) = 0 we used the identity div $\alpha\mathbf{u}$ =α div $\mathbf{u}$ + ($\mathbf{u}$, grad α) and obtain ε div **E** + (**E**, grad ε) = 0. Expressing div **E** from the last equality, we substitute the result into (1.24), writing the wave equation [1] for the strength of the electrical field in an inhomogeneous dielectric medium:

$$\nabla^2\mathbf{E} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \left[\mathrm{grad}\left(\ln\mu\right), \mathrm{rot}\,\mathbf{E}\right] + \mathrm{grad}\left(\mathbf{E}, \mathrm{grad}\left(\ln\varepsilon\right)\right) = 0 \tag{1.25}$$

The same procedure is used for deriving the wave equation for the strength of the magnetic field **H**:

$$\nabla^2\mathbf{H} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} + \left[\mathrm{grad}\left(\ln\varepsilon\right), \mathrm{rot}\,\mathbf{H}\right] + \mathrm{grad}\left(\mathbf{H}, \mathrm{grad}\left(\ln\mu\right)\right) = 0 \tag{1.26}$$

For a homogeneous medium, electrical and magnetic μ permittivitties are constant and the wave equations take the form:

$$\nabla^2\mathbf{E} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \tag{1.27}$$

$$\nabla^2\mathbf{H} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = 0 \tag{1.28}$$

1.2.2. Helmholtz equations

The wave equations, written for the complex amplitudes (monochromatic waves), are referred to as the Helmholtz equations. For an inhomogeneous medium, these equations have the form:

$$\nabla^2\mathbf{E} + k_0^2\varepsilon\mu\mathbf{E} + \left[\mathrm{grad}\left(\ln\mu\right), \mathrm{rot}\,\mathbf{E}\right] + \mathrm{grad}\left(\mathbf{E}, \mathrm{grad}\left(\ln\varepsilon\right)\right) = 0 \tag{1.29}$$

$$\nabla^2\mathbf{H} + k_0^2\varepsilon\mu\mathbf{H} + \left[\mathrm{grad}\left(\ln\varepsilon\right), \mathrm{rot}\mathbf{H}\right] + \mathrm{grad}\left(\mathbf{H}, \mathrm{grad}\left(\ln\mu\right)\right) = 0 \tag{1.30}$$

and for a homogeneous medium:

$$\nabla^2\mathbf{E} + k_0^2\varepsilon\mu\mathbf{E} = 0 \tag{1.31}$$

$$\nabla^2\mathbf{H} + k_0^2\varepsilon\mu\mathbf{H} = 0 \tag{1.32}$$

The equations (1.31) and (1.32) can be solved independently for every projection of the strength of the electrical and magnetic fields **E** and **H**, and these projections can be denoted by a single scalar function U:

$$\nabla^2 U + k_0^2 \varepsilon\mu U = 0 \tag{1.33}$$

1.2.3. The Fock–Leontovich equation

The function U is represented in the form $U = U \exp(ik_0 z)$ and the function is substituted into equation (1.33) for vacuum. Assuming that $\left|\frac{\partial^2 U}{\partial z^2}\right| << k_0 \left|\frac{\partial U}{\partial z}\right|$, we obtain the Fock-Leontovich parabolic wave equation

$$2ik_0 \frac{\partial U}{\partial z} + \Delta_\perp U = 0 \tag{1.34}$$

where $\Delta_\perp U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}$.

The parabolic equation (1.34) in the scalar diffraction theory is used for describing light fields in a paraxial region which propagate mainly along some direction in the space in a small body angle.

1.2.4. Eikonal and transfer equations

The function U is written in the form $U = U_0 \exp(ik_0\psi)$, where $\psi = \psi(x, y, z)$ is the eikonal, U_0 is the amplitude (real function). Substituting this function into equation (1.33) we obtain

$$\frac{\partial^2 U_0}{\partial x^2} + \frac{\partial U^2{}_0}{\partial y^2} + \frac{\partial^2 U_0}{\partial z^2} + 2ik_0\left(\frac{\partial\psi}{\partial x}\frac{\partial U_0}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial U_0}{\partial y} + \frac{\partial\psi}{\partial z}\frac{\partial U_0}{\partial z}\right) +$$

$$+ik_0 U_0\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) - k_0^2 U_0\left(\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2\right) + k_0^2\varepsilon\mu U_0 = 0$$

Equating the apparent part zero, we obtain the transfer equation:

$$2\left(\frac{\partial\psi}{\partial x}\frac{\partial U_0}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial U_0}{\partial y} + \frac{\partial\psi}{\partial z}\frac{\partial U_0}{\partial z}\right) + U_0\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = 0 \tag{1.35}$$

The remaining terms form the following equation:

$$\frac{1}{k_0^2}\left(\frac{\partial^2 U_0}{\partial x^2} + \frac{\partial U^2{}_0}{\partial y^2} + \frac{\partial^2 U_0}{\partial z^2}\right) - U_0\left(\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2\right) + \varepsilon\mu U_0 = 0$$

from which, setting $\lambda \to 0$ (approximation of geometrical optics), we obtain the eikonal equation: