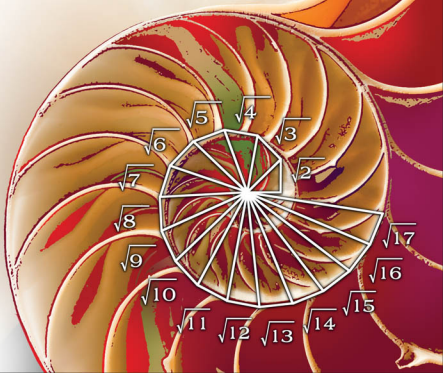


JULIAN HAVIL

THE IRRATIONALS

A Story of the Numbers
You Can't Count On



The Irrationals

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The Irrationals

Julian Havil

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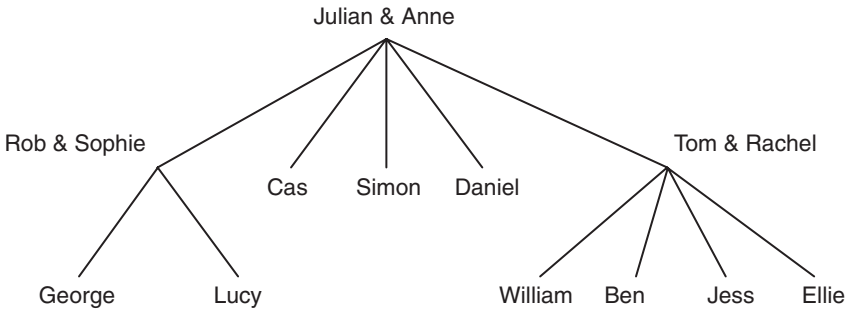
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It was difficult enough being a mathematician, this being the frightening subject of which even educated people knew nothing, not even what it was, and of which they might proudly boast ignorance.

Andrew Hodges (of Alan Turing)

A mathematician learns more and more about less and less, until he knows everything about nothing; whereas a philosopher learns less and less about more and more, until he knows nothing about everything.

Anonymous

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